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ABSTRACT

Radio emission from pulsars is attributed to an instability associated with the creation of electron-positron pairs from gamma rays. The condition for pair creation therefore leads to an "extinction" condition. The relevant physical processes are analyzed in the context of the "PCFB" model, according to which radiation originates at the polar caps and magnetic field lines change from a closed configuration to an open configuration at the "force-balance" or "corotation" radius.

It is found that almost all pulsars with S-type (simple) pulses are in the "RL" regime, in which acceleration is radiation-limited. All pulsars with C- (complex) and D-type (drifting subpulse) pulses are in the complementary "NRL" regime. These pulsars are also close to the extinction condition for a pure dipole model and some pulsars are beyond this condition. In analyzing this model, one may assign a minimum mass to each pulsar in order that the pair-creation condition should be satisfied. This leads, in turn, to an estimate of the minimum surface magnetic-field strength for each pulsar. This value is typically in the range 10^{10} to 10^{11} gauss and has a maximum value of $10^{11.4}$ gauss for the pulsar PSR 2319+60.

Calculations are pursued also for the case of a distorted dipole. Pulsars which should be extinguished according to the pure-dipole model need not be extinguished if the magnetic field is sufficiently distorted at the polar caps. The required distortion seems reasonable, except perhaps for the pulsar PSR 1952+29 for which the required radius of curvature of the magnetic-field lines is comparable with the radius of the polar cap.

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I. Introduction

Beginning with the publications of Radakrishnan and Cooke (1969) and Komesaroff (1970), there has been growing interest in the "polar-cap" model of pulsars, according to which radiation occurs at the magnetic polar caps near the surface of the neutron star. It was shown, some time ago, that many of the properties of radiation from pulsars can be understood in terms of the polar-cap model if one takes account of the annihilation of gamma-rays in a strong magnetic field to produce electron-positron pairs (Sturrock, 1970, 1971a[1]). Three predictions were made in those articles: (a) gamma radiation from the Crab pulsar should be detectable; (b) it should be possible to detect pulsed x-ray emission from the Vela pulsar; and (c) the electric vector of optical radiation from the Crab pulsar should be orthogonal to that of radio emission at the center of a pulse.

Prediction (a) has been confirmed (Apparao, 1969; Browning, et al., 1971; Charman and White, 1970; Albats, et al., 1972) and there is evidence supporting prediction (b) (Moore, et al., 1974; Rappaport, et al., 1974). Prediction (c) was not confirmed, but is now superseded by our more recent analysis of the problem of optical radiation from the Crab pulsar (Sturrock, Petrosian and Turk, 1975).

The above-mentioned analysis of radiation from pulsars was based on a model magnetosphere, closely related to that of Goldreich and

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[1] This article will be referred to briefly as "I"

Julian (1969), according to which the radius of the "Y-type neutral point" R_Y, separating closed field lines from open field lines, coincides with the radius of the light cylinder R_L:

$$R_L = \frac{1}{2\pi}\, cT = 10^{9.7}\, P \tag{1.1}$$

where P (seconds) is the rotation period. This is now referred to as the "PCLC" model.

Subsequent articles (Roberts and Sturrock, 1972a, b, 1973[2]) have called into question the assumption that $R_Y = R_L$. It was shown that one may obtain better agreement with observational data concerning the period-pulse-width distribution, the braking index and interpulses, if one assumes instead that $R_Y = R_{FB}$ where R_{FB} (cm) is the "force-balance" (or "corotation") radius given by

$$R_{FB} = (2\pi)^{-2/3}\, G^{1/3}\, M^{1/3}\, P^{2/3} \approx 10^{-2.9}\, M^{1/3}\, P^{2/3} \tag{1.2}$$

where M (grams) is the mass of the star.

The aim of the present article is to begin a re-investigation of the radiation properties of pulsars, adopting the "PCFB" model rather than the previous PCLC model. The optical radiation from the Crab pulsar has already been discussed (Sturrock, Petrosian and Turk, 1975) according to this model. Our principal concern, in this article, will be a discussion of the condition for pair creation. This leads to an extinction condition, according to which any pulsar will cease to be a radio emitter after its period has increased beyond a certain value. This condition may then be

[2] This article will be referred to briefly as "II".

compared with the period-age distribution of known pulsars, as indicated in a previous article (Sturrock, 1971b).

It will be found that most pulsars meet the extinction condition which may be derived on the basis of a simple model according to which the magnetic field is the same as that of a point dipole located at the center of the star. However, it has recently been found that there are a small number of pulsars which do not satisfy this condition. It is shown that the properties of these pulsars may be understood if one allows for the fact that the magnetic field may depart from the simple model.

Although there has been general agreement from the early history of pulsars (see, for instance, Hewish, 1970) that the period distribution indicates an extinction process, there has been no agreement concerning the nature of this mechanism. In addition to the mechanism which we discuss, the following proposals have been made.

Gunn and Ostriker (1970) interpreted the apparent decrease of radio luminosity with increasing age as attributable to magnetic field decay. Their estimate of the decay constant was [illegible] $\times 10^6$ years. Ruderman and Sutherland (1975) point out that it is probably erroneous to attribute all pulsars to a single evolutionary track. Ruderman (1972) also asserts that the electrical conductivity of typical neutron star matter is so high that it would take far longer than 10^7 years for magnetic field to diffuse out of a neutron star.

Lyne et al. (1975) nevertheless point out that the observed distributions of P and $\dot{P}$ are consistent with a theory in which the magnetic field decays with a time constant of about 10^6 years. It is

their view that the extinction condition is probably related to the magnetic field strength at the velocity of light circle. This suggestion, within the context of their model, would lead to an extinction condition of the form

$$P^2 \tau^{1/2} = \text{const.} \tag{1.3}$$

where τ (seconds) is the "age" defined by

$$\tau = P/\dot{P} \quad . \tag{1.4}$$

With the data presented in Figure 1, this does not seem to represent a particularly sharp boundary to the pulsar distribution. Lyne et al. offer no explanation as to why the radio emission should cease when B_L falls below some critical value, which would have to be of order 1 gauss.

Michel (1975) has recently proposed that radio emission ceases if the gyro-radius of outward-streaming particles is comparable with R_L at the light cylinder. His analysis depends upon the assumption of relativistic space-charge-limited flow, but his formula (which is not derived in his article) for this process appears to be in error. Moreover, his formula for particle energy does not follow from the formulas presented in the appendix of his article.

The formula appropriate for highly relativistic space-charge-limited flow was in fact presented in reference I as equation (I.3.5). This led to formula (I.3.7) as the maximum energy of particles, expressed as an equivalent electrostatic potential. When this estimate is combined with estimates of B_L in the PCLC model, one finds that the radius of

curvature is comparable with R_L for <u>all</u> particles streaming out to the light cylinder, for <u>any</u> values of the period and other parameters, provided that the acceleration is not radiation-limited. On going through a parallel calculation, using formulas given in this article, one finds that the same result is true for the PCFB model. Hence the physical requirement proposed by Michel in fact does not lead to an extinction condition.

Furthermore, the fact that the radius of curvature is comparable with R_L seems to us to be favorable for radio emission rather than unfavorable. The development of electric field parallel to magnetic field requires relative slippage of plasma and magnetic field. Such slippage is in fact achieved when the radius of curvature is large, so that this requirement should promote the development of accelerating electric fields in pulsar magnetospheres, and hence promote the conditions necessary for radio emission.

Ruderman and Sutherland (1975) develop a model which in many ways resembles that of reference I. They derive an extinction condition which, as they themselves remark, is essentially the same as that derived in reference I for the PCLC model. When the extinction line which they propose in their Figure 7 is transformed into our notation, it becomes

$$P \, \tau^{4/5} = 10^{12.4} \quad . \tag{1.5}$$

It is seen from Figure 1 that some pulsars are represented by points well beyond this line.

II. Magnetospheric Structure

To allow for subsequent flexibility in the application of our analysis, we set

$$R_Y = \alpha^{1/3} R_{FB} \quad . \tag{2.1}$$

According to the analysis of II, we expect that $\alpha \approx 1$. Following the analysis of I, modified for the PCFB model of II, we find that the angular radius of the polar cap is given by

$$\theta_p = R^{1/2} R_Y^{-1/2} \approx 10^{1.5} \alpha^{-1/6} M^{-1/6} R^{1/2} P^{-1/3} \quad . \tag{2.2}$$

The magnetic field lines leaving the boundary of this region are tangent to a cone, the half angle ψ_p of which is given by

$$\psi_p = \frac{4}{3} \theta_p \approx 10^{1.6} \alpha^{-1/6} M^{-1/6} R^{1/2} P^{-1/3} \quad . \tag{2.3}$$

The radius of the polar cap is given by

$$R_p = R\theta_p \approx 10^{1.5} \alpha^{-1/6} M^{-1/6} R^{3/2} P^{-1/3} \quad . \tag{2.4}$$

It is seen that the effect of α is that of an apparent change of mass from M to αM.

As in II, we make the simple assumption that

$$\left.\begin{array}{ll} B \propto r^{-3} & R \le r \le R_Y \;, \\ B \propto r^{-2} & R_Y \le r \le R_L \;, \end{array}\right\} \tag{2.5}$$

from which we see that

$$B_L = BR^3 R_Y^{-1} R_L^{-2} \quad . \tag{2.6}$$

In this and subsequent equations, B (without a subscript) denotes the magnetic field strength (in gauss) at the star's magnetic equator. Using the estimate (I.2.6) for the torque Θ on the neutron star,

$$\Theta = \frac{1}{2} R_L^3 B_L^2 \quad , \tag{2.7}$$

we find this to be expressible as

$$\Theta = 10^{-4.1} \alpha^{-2/3} M^{-2/3} B^2 R^6 P^{-7/3} \quad . \tag{2.8}$$

Hence the assumptions underlying this model lead to a value for the braking index $n = 7/3$.

On noting the definition (1.4) for the "age" of a pulsar, we see that

$$\Theta = -I \frac{d\omega}{dt} = 2\pi I P^{-1} \tau^{-1} \tag{2.9}$$

and hence the age is expressible as

$$\tau = 10^{4.9} \alpha^{2/3} M^{2/3} I R^{-6} B^{-2} P^{4/3} \quad , \tag{2.10}$$

where I ($g\ cm^2$) is the moment of inertia of the star. This equation may conveniently be reinterpreted to provide an expression for the surface magnetic field strength in terms of the observable quantities P, τ and the quantities M, I, R characterizing the neutron star:

$$B = 10^{2.5} \alpha^{1/3} M^{1/3} I^{1/2} R^{-3} P^{2/3} \tau^{-1/2} \quad . \tag{2.11}$$

The rate at which rotational energy is being taken from the star may be estimated from equation (2.8):

$$S_T = \omega \Theta = 10^{-3.3} \alpha^{-2/3} M^{-2/3} R^6 B^2 P^{-10/3} \quad . \tag{2.12}$$

However, it may be expressed in terms of the age as

$$S_T = -I\omega \frac{d\omega}{dt} = 10^{1.6} \; I \; P^{-2} \; \tau^{-1} \quad . \tag{2.13}$$

Following (I.2.9), the total current J (emu) flowing through each zone of each polar cap is estimated to be

$$J = \frac{1}{2} B_L R_L = 10^{-7.1} \alpha^{-1/3} M^{-1/3} R^3 B P^{-5/3} \quad . \tag{2.14}$$

The assumption that the current leaving each zone of the polar cap is space-charged limited, and that the electric field is confined to a region of radial extent R_p above the surface, leads to the following estimates for the maximum electric field $\mathcal{E}_M$ (esu) and for the maximum potential change φ_M (esu) across this gap:

$$\mathcal{E}_M = 10^{-7.9} \alpha^{-1/6} M^{-1/6} R^{3/2} B P^{-4/3} \quad , \tag{2.15}$$

$$\varphi_M = 10^{-6.8} \alpha^{-1/3} M^{-1/3} R^3 B P^{-5/3} \quad . \tag{2.16}$$

On noting that the field strength at the poles is 2B, one finds that equation (2.16) is equal to the potential developed between the magnetic pole and the boundary of the polar cap.

In making numerical calculations, we shall adopt the neutron star model computed by Baym, Pethick and Sutherland (1971). This model may be summarized approximately by the expression

$$M = 10^{33.45} \mu \; , \quad I = 10^{44.79} \mu \quad , \quad R = 10^{5.85} \mu^{-1/2} \quad . \tag{2.17}$$

These formulas are accurate at the maximum mass $\mu = 1$, and correct to within 1 dB for masses down to $\mu = 10^{-1.10}$, corresponding to $M = 10^{32.35} = 0.11 \; M_\odot$. Within these limitations, formula (2.11) may conveniently be expressed as

$$B = 10^{18.5} \alpha^{1/3} \mu^{7/3} P^{2/3} \tau^{-1/2} \quad . \tag{2.18}$$

III. Radiation Reaction Limitation

We assume that each polar cap is comprised of two regions, an "electron polar zone" (EPZ) from which electrons stream into the magnetosphere, and an "ion polar zone" (IPZ) from which ions stream into the magnetosphere. The assumption that ions can be emitted from the surface of the neutron star is open to question. Assuming that the magnetic field strength at the surface of a neutron star is 10^{12} gauss or more, Ruderman and Sutherland (1975) argue that ion emission is virtually impossible and construct a detailed model based on this assumption. However, as we shall see, within the context of the PCFB model, there is no evidence that any pulsar requires a surface magnetic field much stronger than 10^{11} gauss, and many may well have field strengths less than 10^{10} gauss. For these low values of the surface magnetic field, the "ultrastrong" regime of Ruderman (1971) appears not to be appropriate.

We therefore consider particles of charge Ze and mass Am_p, where e (esu) is the electron charge and m_p (g) is the proton mass, streaming with energy E (eV) along magnetic field lines with radius of curvature R_c. For an electron, $Z = -1$ and $A = 10^{-3.26}$. Provided the motion is highly relativistic (which is true in the present model), the particle radiates a spectrum peaked at the frequency ν (Hz) given by

$$\nu = 10^{-17.5} A^{-3} E^3 R_c^{-1} \tag{3.1}$$

at a rate S ($\mathrm{erg\ s^{-1}}$) given by

$$S = 10^{-44.2} Z^2 A^{-4} E^4 R_c^{-2} \quad . \tag{3.2}$$

We adopt as R_c the radius of curvature of field lines leaving the edge of the polar cap:

$$R_c = \frac{4}{3} R\theta_p^{-1} = 10^{-1.4} \alpha^{1/6} M^{1/6} R^{1/2} P^{1/3} \quad . \tag{3.3}$$

The energy which a charged particle may acquire in an electric field of strength $\mathcal{E}$ (esu) is limited by radiation reaction. On balancing the driving force to the radiation reaction,

$$eZ\mathcal{E} = c^{-1} S \quad , \tag{3.4}$$

we find that the radiation-limited energy is given by

$$E_{RL} \approx 10^{11.4} Z^{-1/4} A \mathcal{E}^{1/4} R_c^{1/2} \quad . \tag{3.5}$$

Upon using equations (2.15) and (3.3), this may be re-expressed as

$$E_{RL} = 10^{8.7} Z^{-1/4} A \alpha^{1/24} M^{1/24} R^{5/8} B^{1/4} P^{-1/6} \quad . \tag{3.6}$$

On comparing this quantity with $10^{2.5} Z\varphi_M$, the maximum energy (in eV) which a particle of charge Ze will acquire due to a potential φ_M, we see that the acceleration is radiation-limited if

$$P < 10^{-8.7} Z^{5/6} A^{-2/3} \alpha^{-1/4} M^{-1/4} R^{19/12} B^{1/2} \quad . \tag{3.7}$$

We may eliminate the quantity B by using equation (2.11) and hence express the condition for radiation-limited acceleration as follows

$$P_T^{3/8} < 10^{-11.1} Z^{5/4} A^{-1} \alpha^{-1/8} M^{-1/8} I^{3/8} R^{1/8} \quad . \tag{3.8}$$

For the approximate model of equation (2.17), this becomes

$$P_T^{3/8} < 10^{2.2} Z^{5/4} A^{-1} \alpha^{-1/8} \mu^{3/16} \quad \text{for ions} \quad , \tag{3.9}$$

and

$$P_T^{3/8} < 10^{5.5}\,\alpha^{-1/8}\,\mu^{3/16} \quad \text{for electrons} \quad . \tag{3.10}$$

We find from equation (3.9) that radiation reaction of ions is not important for the Crab pulsar or any other pulsar.

For $\alpha \approx 1$ and μ in the range $10^{-1.1}$ to 1, the condition (3.10) is expressible as

$$P_T^{3/8} < 10^{5.4} \tag{3.11}$$

to within 1 dB. Comparison of this condition with pulsar data shows that electron acceleration is radiation-limited for most pulsars.

IV. The Pair-Creation Condition

We see from equation (3.1) that curvature-radiation by electrons will lead to the production of gamma rays of energy

$$E_\gamma = 10^{-22.1} E_e^3 R_c^{-1} \quad . \tag{4.1}$$

If acceleration is limited by radiation reaction ("RL"), the electron energy is given by

$$E_{e,RL} = 10^{5.4} \alpha^{1/24} M^{1/24} R^{5/8} B^{1/4} P^{-1/6} \quad [RL] \tag{4.2}$$

which one obtains from equation (3.6). If acceleration is not limited by radiation-reaction ("NRL"), the electron energy is given by $10^{2.5} \varphi_M$ which is found, from equation (2.16), to be

$$E_{e,M} = 10^{-4.3} \alpha^{-1/3} M^{-1/3} R^3 B P^{-5/3} \quad [NRL] \quad . \tag{4.3}$$

For these two cases, we find that the gamma-ray energy is given by

$$E_\gamma = 10^{-4.5} \alpha^{-1/24} M^{-1/24} R^{11/8} B^{3/4} P^{-5/6} \quad [RL] \tag{4.4}$$

or

$$E_\gamma = 10^{-33.6} \alpha^{-7/6} M^{-7/6} R^{17/2} B^3 P^{-16/3} \quad [NRL] \quad . \tag{4.5}$$

We again use the condition (I.4.7) for pair creation:

$$B_\perp E_\gamma \geq 10^{18.6} \quad . \tag{4.6}$$

The maximum transverse value of the magnetic field strength is given by (I.4.8):

$$B_{\perp M} = 10^{-1.0} B\theta_p \quad . \tag{4.7}$$

Hence, using equations (2.2), (4.4), (4.5) and (4.7), we find that the condition for pair creation at the EPZ is expressible as

$$P \leq P_{ePC} = 10^{-19.4}\ \alpha^{-5/28}\ M^{-5/28}\ R^{45/28}\ B^{3/2} \qquad [\mathrm{RL}] \qquad (4.8)$$

or

$$P \leq P_{ePC} = 10^{-9.1}\ \alpha^{-4/17}\ M^{-4/17}\ R^{27/17}\ B^{12/17} \qquad [\mathrm{NRL}] \quad . \qquad (4.9)$$

On using equation (2.11), we may eliminate B and so obtain the pair creation condition in the following forms*:

$$\tau \leq 10^{-20.9}\ \alpha^{3/7}\ M^{3/7}\ I\ R^{-27/7} \qquad [\mathrm{RL}] \quad , \qquad (4.10)$$

$$P\tau^{2/3} \leq 10^{-14.0}\ I^{2/3}\ R^{-1} \qquad [\mathrm{NRL}] \quad . \qquad (4.11)$$

On using the approximate neutron-star model of equation (2.17), these conditions become

$$\tau \leq 10^{15.7}\ \alpha^{3/7}\ \mu^{47/14} \qquad [\mathrm{RL}] \quad , \qquad (4.12)$$

$$P\tau^{2/3} \leq 10^{10.1}\ \mu^{7/6} \qquad [\mathrm{NRL}] \quad . \qquad (4.13)$$

Equations (4.12) and (4.13) show that, for a fixed value of α, the cut-off condition has a disjointed form in the P-τ plane, as shown in Figure 1, which is constructed for $\alpha = 1$. The "critical" values of P and τ, representing the boundary between RL and NRL conditions as derived from equations (4.12) and (4.13), are found to be

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*If the critical values of τ in equations (4.10) and (4.11) are denoted by τ_{PCR} and τ_{PCN}, respectively, and if the critical value of τ in equation (3.8), evaluated for electrons, is denoted by τ_{RL}, we find that $\tau_{PCN}^{16} = \tau_{PCR}^{7}\ \tau_{RL}^{9}$.

$$P_c = 10^{-0.4}\,\alpha^{-2/7}\,\mu^{-15/14} \qquad \tau_c = 10^{15.7}\,\alpha^{3/7}\,\mu^{47/14} \quad . \tag{4.14}$$

The line traced by P_c, τ_c, for $\alpha = 1$ and varying μ, is close to the approximate condition (3.11).

One may also invert these equations to find the minimum mass of a star consistent with the pair-creation requirement. If μ_m is the minimum value of μ, we find that

$$\mu_m = 10^{-4.8}\,\alpha^{-6/47}\,\tau^{14/47} \quad [\mathrm{RL}] \quad , \tag{4.15}$$

$$\mu_m = 10^{-8.7}\,P^{6/7}\,\tau^{4/7} \quad [\mathrm{NRL}] \quad . \tag{4.16}$$

On using equation (2.18), we may also obtain estimates of the minimum magnetic field strength B_m, consistent with pair-creation:

$$B_m = 10^{7.3}\,\alpha^{5/141}\,P^{2/3}\,\tau^{55/282} \quad [\mathrm{RL}] \quad , \tag{4.17}$$

$$B_m = 10^{-1.8}\,\alpha^{1/3}\,P^{8/3}\,\tau^{5/6} \quad [\mathrm{NRL}] \quad . \tag{4.18}$$

These values are shown in Figure 1. We see that $B_m < 10^{10.4}$ for all pulsars for which the acceleration is radiation limited. Also $10^{10.4} < B_m < 10^{11}$ for most remaining pulsars. It is interesting that the lines $\log B_m = \mathrm{const}$ are more closely spaced above the RL line than below it.

When these calculations are repeated for ions instead of electrons (noting that radiation reaction is unimportant), we find that (4.9) is replaced by

$$P \le P_{i\,PC} = 10^{-10.9}\,Z^{9/17}\,A^{-9/17}\,\alpha^{-4/17}\,M^{-4/17}\,R^{27/17}\,B^{12/17} \tag{4.19}$$

so that (4.11) and (4.13) are replaced by

$$P\tau^{2/3} \leq 10^{-17.2}\ Z\ A^{-1}\ I^{2/3}\ R^{-1} \tag{4.20}$$

and

$$P\tau^{2/3} \leq 10^{6.8}\ Z\ A^{-1}\ \mu^{7/6} \tag{4.21}$$

respectively. We see that the Crab pulsar ($P = 10^{-1.48}$, $\tau = 10^{10.90}$) will give rise to pair creation at the IPZ provided that $\mu > 10^{-0.9}$ for protons, or $\mu > 10^{-0.6}$ for other ions, (fully stripped, with $A/Z \approx 2$). According to the present model, there will be no pair creation at the IPZ of the Vela pulsar ($P = 10^{-1.05}$, $\tau = 10^{11.86}$) for any value of μ in the range $10^{-1.1} \leq 1$.

According to our previous theory (I), pair creation leads to an electromagnetic instability of the two-stream type in the polar cap regions. This in turn leads to bunching, which results in radio emission by coherent curvature radiation. Hence the condition for pair creation is also the condition for radio emission. The limiting condition (given by the equalities in equations (4.10), (4.11), etc.) therefore determines the onset of pulsar "extinction".

V. Distorted Magnetospheres

When we come to compare pulsar data with the extinction condition derived in the last section (as we shall do in Section VI), we find that certain pulsars are clearly beyond the extinction condition. Possible resolutions of this discrepancy will be discussed in Section VI. One of the possibilities is that the magnetosphere differs substantially from that of a simple dipole. For this reason, we estimate in this section the amount of distortion which would be required to permit pair-production in a pulsar when this would not occur in a simple dipole field.

We now parameterize the curvature of the magnetic field lines by rewriting equation (3.3) as

$$R_c = \eta \frac{4}{3} R \, \theta_p^{-1} = \eta \, 10^{-1.4} \, \alpha^{1/6} \, M^{1/6} \, R^{1/2} \, P^{1/3} \quad , \qquad (5.1)$$

so that $\eta = 1$ is the previous undistorted dipole model. We may now repeat the calculations of Sections III and IV. The radiation-reaction limitation is expressed as

$$P\tau^{3/8} \, \eta^{1/2} < 10^{-11.1} \, Z^{5/4} \, A^{-1} \, \alpha^{-1/8} \, M^{-1/8} \, I^{3/8} \, R^{1/8} \quad . \qquad (5.2)$$

When we use the model of equation (2.17), this becomes

$$P\tau^{3/8} \, \eta^{1/2} < 10^{5.5} \, \alpha^{-1/8} \, \mu^{3/16} \quad \text{for electrons.} \qquad (5.3)$$

The pair-creation condition [equations (4.10) and (4.11)] becomes

$$\tau \leq 10^{-20.9} \, \alpha^{3/7} \, M^{3/7} \, I \, R^{-27/7} \, \eta^{-4/7} \quad [\mathrm{RL}] \, , \qquad (5.4)$$

$$P\tau^{2/3} \leq 10^{-14.0} \, I^{2/3} \, R^{-1} \, \eta^{-2/3} \quad [\mathrm{NRL}] \quad . \qquad (5.5)$$

We shall investigate the distorted-dipole model only for neutron stars of maximum mass, i.e. for $\mu = 1$. Then equations (5.4) and (5.5) become

$$\tau \le 10^{15.7}\ \eta^{-4/7} \quad [\mathrm{RL}]\ , \tag{5.6}$$

$$P\tau^{2/3} \le 10^{10.1}\ \eta^{-2/3} \quad [\mathrm{NRL}]\ . \tag{5.7}$$

We may obtain the condition separating radiation-reaction-limited (RL) and non-radiation-reaction-limited (NRL) acceleration by eliminating η from these equations. This gives the separatrix as

$$P_c\ \tau_c^{-1/2} = 10^{-8.2}\ . \tag{5.8}$$

The results of these calculations are shown in Figure 2, which also shows values of the magnetic-field strength appropriate for this model, as calculated from equation (2.18). It also shows the RL-NRL separatrix for the undistorted dipole, but for a neutron star of maximum mass, as calculated from equation (3.10), and the limiting condition for pair creation at the IPZ, as calculated from equation (4.20).

In order to gauge the extent of the distortion required for pair-creation, we have included in Figure 2 lines corresponding to the conditions $R_c = R$ and $R_c = R_p$. The former indicates substantial distortion from a pure dipole configuration, and the latter would seem to represent the maximum distortion consistent with the model on which our calculations are based.

VI. Discussion

The results of these calculations, and comparison with observational data, are summarized in Figures 1 and 2. We see from Figure 1 that most pulsars satisfy or nearly satisfy the pair-creation condition for undistorted dipoles. The clear exceptions are PSR 0138+59, PSR 0809+74, PSR 1730-22, PSR 1819-22, PSR 1943+18, PSR 1944+17 and PSR 1952+29. These will be discussed later.

We see from Figure 1 or from equation (3.10) that most pulsars seem to satisfy the radiation-limited condition. We particularly note the distribution of pulsars in Figure 1 according to pulse shape (Huguenin, Manchester and Taylor, 1971; Taylor and Huguenin, 1971; Taylor and Manchester, 1975). All S-type (simple) pulsars (except PSR 1541+09) satisfy this condition and also satisfy the pair-creation condition. On the other hand, all C- and D-type (complex and drifting subpulse) pulsars seem not to be radiation limited. Of these two groups, the C-type pulsars, [with one exception (PSR 1237+25)] satisfy the pair-creation condition for undistorted dipoles. On the other hand, D-type pulsars are either close to the limiting pair-creation condition or, in one case (PSR 0809+74), well beyond that limit. We further see from Figure 1 that the curves B_m = constant are almost parallel to the RL line. Hence, for pulsars compatible with the undistorted-dipole hypothesis, all S-type pulsars (except PSR 1541+09) require magnetic fields no higher than $10^{10.4}$ gauss, whereas all C- and D-type pulsars require fields in excess of $10^{10.4}$ gauss. However, few require fields higher than 10^{11} gauss. The highest is PSR 2319+60 which requires a magnetic field of $10^{11.4}$ gauss.

The following table lists those pulsars which we regard as incompatible with the undistorted-dipole hypothesis. Some of these require only modest distortion of the magnetic field, but pulsars PSR 1730-22, PSR 1943+18 and PSR 1952+29 require highly distorted magnetic fields. Since one must expect some departure from pure dipole geometry in the magnetic field patterns of pulsars, it is not unreasonable that a few pulsars should require substantial distortion to explain their properties. Whether the distortion of pulsars required in our model for pair creation is in fact reasonable is a question which we do not feel we can answer.

Although it seems that the characteristics of pulsars with very high ages can be explained on the basis of the distorted-dipole model, it is nevertheless worthwhile to ask whether there are any other ways to reconcile the properties of these pulsars with the pulsar model of this article. We can see the following possibilities:

If there is a steady transfer of material from the force-balance region to the surface of the star (on a transient basis), this will represent a steady reduction in the moment of inertia of the system. This process will tend to "spin up" the star thus increasing the age. It is even possible that this transient spin-up effect may exceed the magnetic torque, resulting in a negative age for the pulsar. There is some indication (Lyne et al., 1975) that PSR 1813-26 may have negative "age", although the measurement errors are still too large to definitely determine the sign.

If, as a result of accretion, there is a dense plasma in the neighborhood of the pulsar (at or beyond the light-cylinder radius),

there may be an inflow of ions on the same field lines which carry an outflow of electrons. In this case, there could be a two-stream instability, leading to radio emission, without the necessity of pair creation.

Another possibility is that some neutron stars are substantially more massive than the maximum mass permitted by the Baym, Pethick, Sutherland (1971) model. In this case, some of the pulsars under consideration may exhibit pair creation without substantial distortion or with only slight distortion of the magnetic field. One may also note the possibility that the magnetospheric structure contains oscillatory components, possibly representing torsional oscillations of the magnetic field. In this case, the maximum value of the electric field at the polar caps may be substantially larger than the average value, leading to pair creation under conditions ruled out in the simple steady-state model. Such behavior might possibly be related to some form of the drifting-subpulse phenomenon (Backer, 1973).

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TABLE 1

Name	RL/NRL	P	log P	log τ	log η	log R_c
0138+59	NRL	1.223	.09	15.85	- .89	6.21
0301+19	NRL	1.388	.14	15.03	- .15	6.95
0809+74	NRL	1.292	.11	15.91	- .98	6.12
1112+50	NRL	1.656	.22	14.81	- .04	7.06
1237+25	NRL	1.382	.14	15.16	- .28	6.82
1700-32	NRL	1.212	.08	15.25	- .28	6.82
1730-22	RL	0.872	-.06	16.58	-1.57	5.53
1819-22	NRL	1.874	.27	15.51	- .83	6.27
1857-26	NRL	0.612	-.21	15.58	- .17	6.93
1919+21	NRL	1.337	.13	15.00	- .10	7.00
1943+18	RL	1.069	.03	16.64	-1.69	5.41
1944+17	RL	0.441	-.36	16.26	-1.01	6.09
1952+29	RL	0.427	-.37	17.10	-2.49	4.61
2106+44	RL	0.415	-.38	15.86	- .31	6.79
2111+46	NRL	1.015	.01	15.15	- .07	7.03
2305+55	NRL	0.475	-.32	15.84	- .26	6.84

Table 1: The minimum distortion required for pair production is characterized, in the sixth column, by the parameter η and, in the seventh column, by the radius of curvature R_c.

FIGURE CAPTIONS

Figure 1. Distribution of pulsars according to log P and log τ. The extinction condition (lines of μ_m = const.) and lines of minimum magnetic field strength are plotted assuming the undistorted dipole model ($\eta = 1$). The NRL-RL line divides the pulsars in two groups -- radiation-reaction-limited and non-radiation-reaction-limited acceleration.

Figure 2. Distribution of pulsars according to log P and log τ. The extinction condition for different values of the distortion parameter η are plotted assuming $\mu = 1$ (maximum mass). The lines of constant magnetic field strength are also plotted under the assumption $\mu = 1$. The IPZ extinction lines are also plotted for protons ($A/Z = 1$) and for high mass ions ($A/Z = 2$). Note that only the Crab pulsar can be producing pairs through ion acceleration.

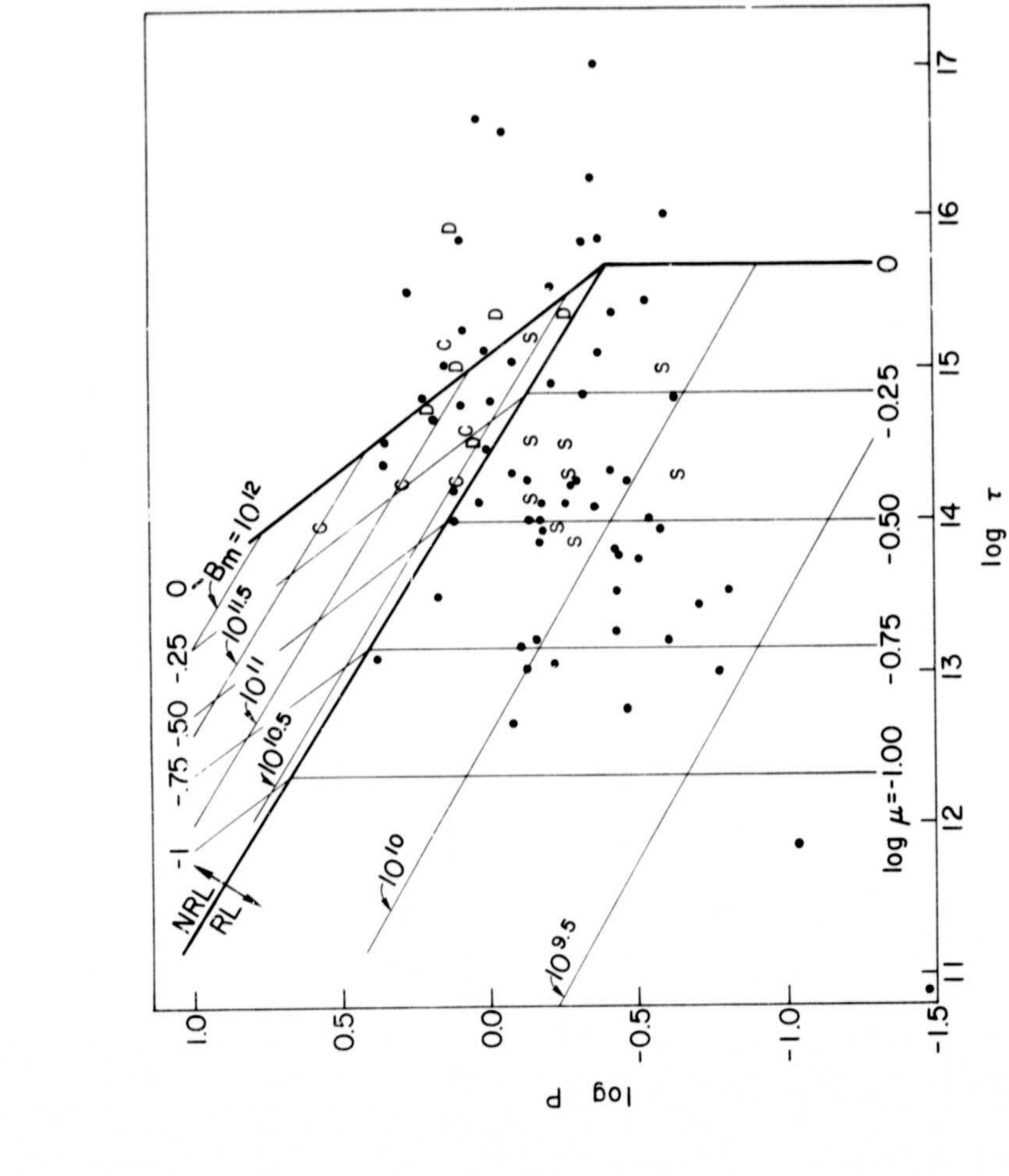
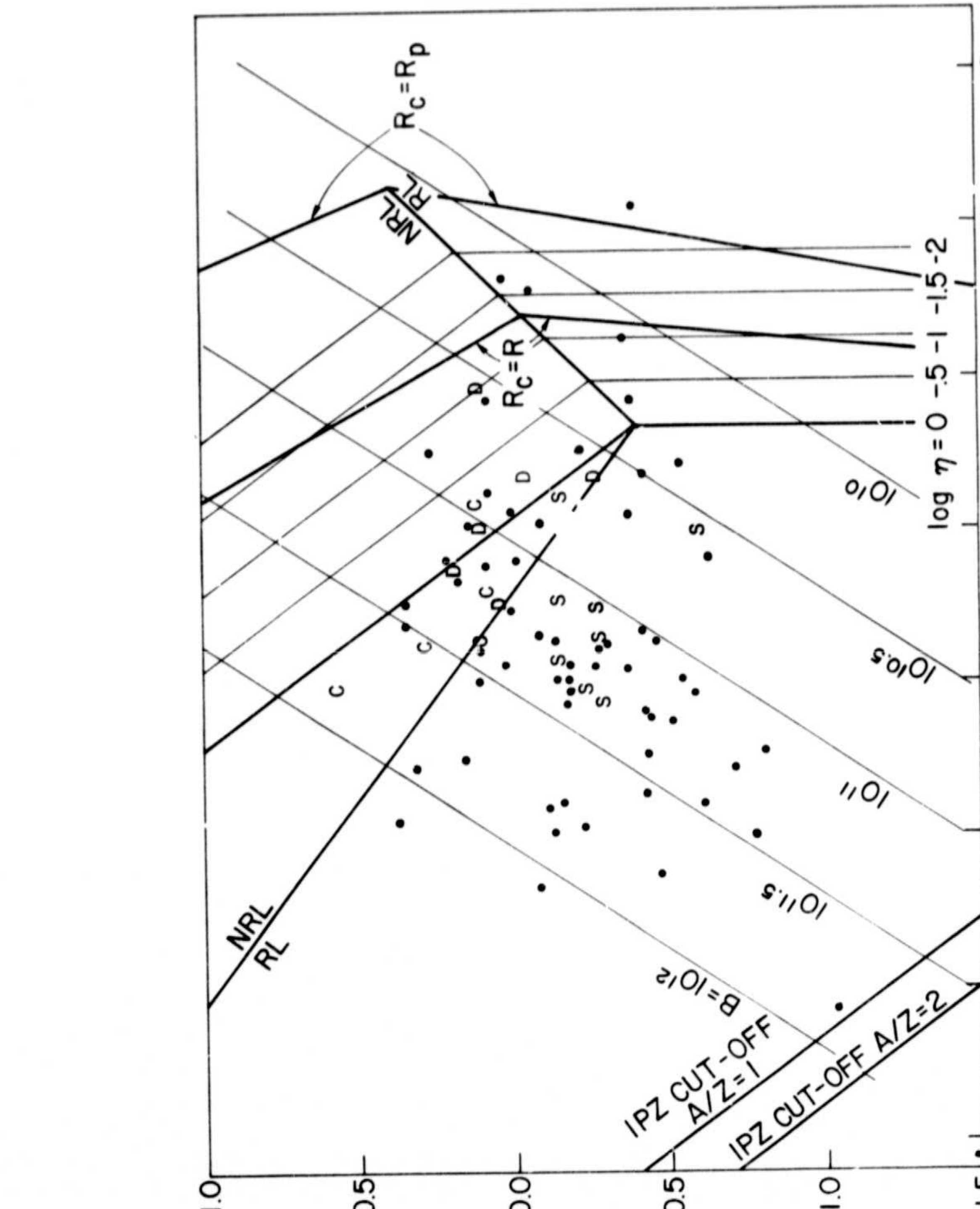

log P
log τ
log μ=-1.00
-0.75
-0.50
-0.25
0
Bm=10^12
10^11.5
10^11
10^10.5
10^10
10^9.5
NRL
RL
-1 -.75 -.50 -.25 0
1.0
0.5
0.0
-0.5
-1.0
-1.5
11
12
13
14
15
16
17

Figure 1



log P
log τ
1.0
0.5
0.0
-0.5
-1.0
-1.5
11
12
13
14
15
16
17
18
NRL
RL
$R_c=R_p$
$R_c=R$
log η = 0 -.5 -1 -1.5 -2
10^{10}
$10^{10.5}$
10^{11}
$10^{11.5}$
$B=10^{12}$
IPZ CUT-OFF A/Z=1
IPZ CUT-OFF A/Z=2

Figure 2